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On the cancelation of quantum-mechanical corrections at the periodic motion

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Abstract

The paper contains description of the path integrals in the action-angle phase space. It allows to split the action and angle quantum degrees of freedom and to show that the angular quantum corrections are cancel each other if the classical trajectory is periodic. The considered in the paper example shows that the quantum problem can be quasiclassical over the part (angular in the considered case) degrees of freedom.

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1 Introduction

It will be shown in this paper that the quantum fluctuations of angular variables cancel each other if the classical motion is periodic. This cancelation mechanism can be used for the path-integral explanation of the rigid rotator problems integrability (last one is isomorphic to the Pocshle-Teller problem [1]) [2]. Note also that the classical trajectories of all known integrable quantum-mechanical problems (of the rigid rotator, of the H-atom [3], etc.) are periodic.

Our technical problem consist in necessity to extract the quantum angular degrees of freedom. For this purpose we will use the unitary definition of the path integral measure which guarantees the conservation of total probability at arbitrary transformations of the path integral variables [4]. It will allow to define the path integral in the phase space of action-angle variables and, correspondingly, to define the quantum measure of the angular degrees of freedom.

Mostly probable that the considered phenomena has the general character and its demonstration will be fruitful. For simplicity this effect of cancellations we will demonstrate on the one-dimensional λx^4 model [5]. In the following section the brief description of unitary definition of the path-integral measure will be given. The perturbation theory in terms of action-angle variables will be constracted in Sec.3 (the scheme of transformed perturbation theory was given in [4]). In Sec.4 the cancelation mechanism will be demonstrated.

2 The unitary definition of the path-integral measure

We will calculate the the probability

$$R(E) = \int dx_1 dx_2 |A(x_1, x_2; E)|^2, \tag{2.1}$$

to introduce the unitary definition of path-integral measure [6]. Here

$$A(x_1, x_2; E) = i \int_0^\infty dT e^{iET} \int_{x(0)=x_1}^{x(T)=x_2} Dx e^{iS_{C_+(T)}(x)} \tag{2.2}$$

is the amplitude. The action

$$S_{C_+(T)}(x) = \int_{C_+(T)} dt (\frac{1}{2}\dot{x}^2 - \frac{\omega_0^2}{2}x^2 - \frac{\lambda}{4}x^4) \tag{2.3}$$

is defined on the Mills' contour [7]:

$$C_\pm(T) : t \to t \pm i\epsilon, \quad \epsilon \to +0, \quad 0 \le t \le T. \tag{2.4}$$

So, we will omit the calculation of the amplitude since it is sufficient to now $R(E)$ for the bound states energies computation (see also [8] where a many-particles system was considered from this point of view).

Inserting (2.2) into (2.1) we find:

$$R(E)=\int_0^\infty dT_+dT_-e^{iE(T_+-T_-)}\int_{x_+(0)=x_-(0_-)}^{x_+(T_+)=x_-(T_-)}Dxe^{iS_{C^-}(x)} \tag{2.5}$$

is described by the closed-path integral. The total action

$$S_{C^-}(x)=S_{C_+(T_+)}(x_+)-S_{C_-(T_-)}(x_-), \tag{2.6}$$

where the integration over turning points

$$x_1=x_+(0)=x_-(0),\quad x_2=x_+(T_+)=x_-(T_-) \tag{2.7}$$

was performed.

Using the linear transformations:

$$x_\pm(t)=x(t)\pm e(t) \tag{2.8}$$

and

$$T_\pm=T\pm\tau \tag{2.9}$$

we find, calculating integrals over $e(t)$ and τ perturbatively [4], that

$$R(E)=2\pi\int_0^\infty dT\exp\{\frac{1}{2i}\hat\omega\hat\tau-i\int_{C^{(+)}(T)}dt\hat{j}(t)\hat{e}(t)\}\int Dxe^{-i\tilde{H}(x;\tau)-iV_T(x,e)}\times$$
$$\times\delta(E+\omega-H_T(x))\prod_t\delta(\ddot{x}+\omega_0^2x+\lambda x^3-j). \tag{2.10}$$

The "hat" symbol means differentiation over corresponding auxiliary quantity. For instance,

$$\hat\omega\equiv\frac{\partial}{\partial\omega},\quad \hat{j}(t)=\frac{\delta}{\delta j(t)}. \tag{2.11}$$

It will be assumed that

$$\begin{aligned}\hat{j}(t\in C_\pm)j(t'\in C_\pm)&=\delta(t-t'),\\ \hat{j}(t\in C_\pm)j(t'\in C_\mp)&\equiv 0.\end{aligned} \tag{2.12}$$

The time integral over contour $C^{(\pm)}(T)$ means that

$$\int_{C^{(\pm)}(T)}=\int_{C_+(T)}\pm\int_{C_-(T)}. \tag{2.13}$$

At the end of calculations the limit $(\omega,\tau,j,e)=0$ must be calculated. The explicit form of $\tilde{H}(x;\tau)$, $V_T(x,e)$ will be given later; $H_T(x)$ is the Haniltonian at the time moment $t=T$.

The functional δ-function unambiguously determines the contributions in the path integral. For this purpose we must find the strict solution $x_j(t)$ of the equation of motion:

$$\ddot{x}+\omega_0^2x+\lambda x^3-j=0, \tag{2.14}$$

expanding it over j. In zero order over j we have the classical trajectory x_c which is defined by the equation of motion:

$$\ddot{x} + \omega_0^2 x + \lambda x^3 = 0. \tag{2.15}$$

This equation is equivalent to the following one:

$$t + \theta_0 = \int^x dx \{2(h - \omega_0^2 x^2 - \lambda x^4)\}^{-1/2}. \tag{2.16}$$

The solution of this equation is the periodic elliptic function [9]. Here (h, θ_0) are the constants of integration of eq.(2.15).

The mapping of our problem on the action-angle phase space will be performed using representation (2.10) [4]. Using the obvious definition of the action:

$$I = \frac{1}{2\pi} \oint \{2(h - \omega_0^2 x^2 - \lambda x^4)\}^{1/2}, \tag{2.17}$$

and of the angle

$$\phi = \frac{\partial h}{\partial I} \int^{x_c} \{2(h - \omega_0^2 x^2 - \lambda x^4)\}^{-1/2} \tag{2.18}$$

variables [11] we easily find from (2.10) that

$$R(E) = 2\pi \int_0^\infty dT \exp\{\frac{1}{2i}\hat{\omega}\hat{\tau} - i\int_{C^{(+)}(T)} dt \hat{j}(t)\hat{e}(t)\} \int DID\phi e^{-i\tilde{H}(x_c;\tau) - iV_T(x_c, e)} \times \\ \times \delta(E + \omega - h_T(I)) \prod_t \delta(\dot{I} - j\frac{\partial x_c}{\partial \phi})\delta(\dot{\phi} - \Omega(I) + j\frac{\partial x_c}{\partial I}), \tag{2.19}$$

where $x_c = x_c(I, \phi)$ is the solution of eq.(2.18) with $h = h(I)$ as the solution of eq.(2.17) and the frequency

$$\Omega(I) = \frac{\partial h}{\partial I}. \tag{2.20}$$

Representation (2.19) is not the full solution of our problem: the action and angle variables are still interdependent since they both are exited by the same source $j(t)$. This reflects the Lagrange nature of the path-integral description of (x, p) phase-space motion. The true Hamiltone's description must contain independent quantum sources of action and angle variables.

3 The perturbation theory in the action-angle phase space

The structure of source terms $j\partial x_c/\partial\phi$ and $j\partial x_c/\partial I$ shows that the source of quantum fluctuations is the classical trajectories perturbations and j is the auxiliary variable. It

allows to regroup the perturbation series in a following manner. Let us consider the action of the perturbation-generating operators:

$$e^{-i\int_{C^{(+)}(T)}dt\hat{j}(t)\hat{e}(t)}e^{-iV_T(x,e)}\prod_t\delta(\dot{I}+j\frac{\partial x_c}{\partial\phi})\delta(\dot{\phi}-\Omega(I)-j\frac{\partial x_c}{\partial I}=$$
$$=\int De_I De_\phi e^{i\int_{C^{(+)}}dt(e_I\dot{I}+e_\phi(\dot{\phi}-\Omega(I)))}e^{-iV_T(x,e_c)}, \tag{3.1}$$

where

$$e_c(e_I,e_\phi)=e_I\frac{\partial x_c}{\partial\phi}-e_\phi\frac{\partial x_c}{\partial I} \tag{3.2}$$

The integrals over (e_I,e_ϕ) will be calculated perturbatively:

$$e^{-iV_T(x,e_c)}=\sum_{n_I,n_\phi=0}^{\infty}\frac{1}{n_I!n_\phi!}\int\prod_{k=1}^{n_I}(dt_k e_I(t_k))\prod_{k=1}^{n_\phi}(dt'_k e_\phi(t'_k))P_{n_I,n_\phi}(x_c,t_1,...,t_{n_I},t'_1,...,t_{n_\phi}), \tag{3.3}$$

where

$$P_{n_I,n_\phi}(x_c,t_1,...,t_{n_I},t'_1,...,t_{n_\phi})=\prod_{k=1}^{n_I}\hat{e}'_I(t_k)\prod_{k=1}^{n_\phi}\hat{e}'_\phi(t'_k)e^{-iV_T(x,e'_c)}. \tag{3.4}$$

Here $e'_c\equiv e_c(e'_I,e'_\phi)$ and the derivatives in this equality are calculated at $e'_I=0$, $e'_\phi=0$. At the same time,

$$\prod_{k=1}^{n_I}e_I(t_k)\prod_{k=1}^{n_\phi}e_\phi(t'_k)=\prod_{k=1}^{n_I}(i\hat{j}_I(t_k))\prod_{k=1}^{n_\phi}(i\hat{j}_\phi(t'_k))e^{-i\int_{C^{(+)}}dt(j_I(t)e_I(t)+j_\phi(t)e_\phi(t))}. \tag{3.5}$$

The limit $(j_I,j_\phi)=0$ is assumed. Inserting (3.3), (3.5) into (3.1) we will find new representation for $R(E)$:

$$R(E)=2\pi\int_0^\infty dT\exp\{\frac{1}{2i}\hat{\omega}\hat{\tau}-i\int_{C^{(+)}(T)}dt(\hat{j}_I(t)\hat{e}_I(t)+\hat{j}_\phi(t)\hat{e}_\phi(t))\}\times$$
$$\times\int DID\phi e^{-i\tilde{H}(x_c;\tau)-iV_T(x_c,e_c)}\delta(E+\omega-h_T(I))\prod_t\delta(\dot{I}-j_I)\delta(\dot{\phi}-\Omega(I)-j_\phi) \tag{3.6}$$

in which the action and the angle degrees of freedom are decoupled.

Solving the canonical equations of motion:

$$\dot{I}=j_I,\quad \dot{\phi}=\Omega(I)+j_\phi, \tag{3.7}$$

the boundary conditions:

$$I_j(0)=I_0,\quad \phi_j(0)=\phi_0 \tag{3.8}$$

for the solutions I_j,ϕ_j of eqs.(3.7) will be used. This will lead to the following Green function:

$$g(t-t')=\Theta(t-t'), \tag{3.9}$$

with symmetric step function: $\Theta(0) = 1/2$. The solutions of eqs.(3.7) have the form:

$$I_j(t) = I_0 + \int dt' g(t-t') j_I(t') \equiv I_0 + I'(t),$$
$$\phi_j(t) = \phi_0 + \tilde{\Omega}(I_j)t + \int dt' g(t-t') j_\phi(t') \equiv \phi_0 + \tilde{\Omega}(I_j)t + \phi'(t), \tag{3.10}$$

where

$$\tilde{\Omega}(I_j) = \frac{1}{t} \int dt' g(t-t') \Omega(I_0 + I'(t')). \tag{3.11}$$

Inserting (3.10) into (3.6) we find:

$$R(E) = 2\pi \int_0^\infty dT \exp\{\frac{1}{2i}\hat{\omega}\hat{\tau} - i\int_{C^{(+)}(T)} dt(\hat{j}_I(t)\hat{e}_I(t) + \hat{j}_\phi(t)\hat{e}_\phi(t))\} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} d\phi_0 e^{-i\tilde{H}(x_c;\tau) - iV_T(x_c,e_c)} \delta(E + \omega - h_T(I_j)), \tag{3.12}$$

where

$$x_c = x_c(I_j, \phi_j) = x_c(I_0 + I'(t), \phi_0 + \tilde{\Omega}(I_j)t + \phi'(t)) \tag{3.13}$$

and e_c was defined in (3.2). Note that the measure of the integrals over (I_0, ϕ_0) was defined without of the Faddeev-Popov's ansatz [10] and there is not any "hosts" since the Jacobian of transformation is equal to one.

We can extract the Green function into the perturbation-generating operator using the equalities:

$$\hat{j}_I(t) = \int dt' g(t-t') \hat{I}'(t),$$
$$\hat{j}_\phi = \int dt' g(t-t') \hat{\phi}'(t), \tag{3.14}$$

which evidently follows from (3.10). In result,

$$R(E) = 2\pi \int_0^\infty dT \exp\{\frac{1}{2i}\hat{\omega}\hat{\tau} - i\int_{C^{(+)}(T)} dtdt' g(t'-t)(\hat{I}(t)\hat{e}_I(t') + \hat{\phi}(t)\hat{e}_\phi(t'))\} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} d\phi_0 e^{-i\tilde{H}(x_c;\tau) - iV_T(x_c,e_c)} \times$$
$$\times \delta(E + \omega - h_T(I_j)), \tag{3.15}$$

where x_c was defined in (3.13).

We can define the formalism without doubling of degrees of freedom. One can use the fact that the action of perturbation-generating operators and the analytical continuation to the real times are the commuting operations. This can be seen easily using the definition (5.1). In result:

$$R(E) = 2\pi \int_0^\infty dT \exp\{\frac{1}{2i}\hat{\omega}\hat{\tau} - i\int_0^T dtdt' \Theta(t'-t)(\hat{I}(t)\hat{e}_I(t') + \hat{\phi}(t)\hat{e}_\phi(t'))\} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} d\phi_0 e^{-i\tilde{H}(x_c;\tau) - iV_T(x_c,e_c)} \delta(E + \omega - h_T(I_0 + I(T)), \tag{3.16}$$

where

$$\tilde{H}_T(x_c;\tau) = 2\sum_{n=1}^{\infty} \frac{\tau^{2n+1}}{(2n+1)!}\frac{d^{2n}}{dT^{2n}} h(I_0 + I(T)) \tag{3.17}$$

and

$$-V_T(x_c, e_c) = S(x_c + e_c) - S(x_c - e_c). \tag{3.18}$$

Now we will use the last δ-function:

$$R(E) = 2\pi \int_0^\infty dT \exp\{\frac{1}{2i}(\hat{\omega}\hat{\tau} + \int_0^T dtdt'\Theta(t'-t)(\hat{I}(t)\hat{e}_I(t') + \hat{\phi}(t)\hat{e}_\phi(t'))\} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} \frac{d\phi_0}{\Omega(E+\omega)} e^{-i\tilde{H}(x_c;\tau) - iV_T(x_c,e_c)}, \tag{3.19}$$

Here

$$x_c(t) = x_c(I(E+\omega) + I(t) - I(T), \phi_0 + \tilde{\Omega}t + \phi(t)). \tag{3.20}$$

Eq.(3.19) contains unnecessary contributions: the action of the operator

$$\int_0^T dtdt'\Theta(t-t')\hat{e}_I(t)\hat{I}(t') \tag{3.21}$$

on $\tilde{H}_T$, defined in (3.17), leads to the time integrals with zero integration range:

$$\int_0^T dt\Theta(T-t)\Theta(t-T) = 0. \tag{3.22}$$

Using this fact,

$$R(E) = 2\pi \int_0^\infty dT e^{\frac{1}{2i}\int_0^T dtdt'\Theta(t'-t)(\hat{I}(t)\hat{e}_I(t') + \hat{\phi}(t)\hat{e}_\phi(t'))} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} \frac{d\phi_0}{\Omega(E)} e^{-iV_T(x_c,e_c)}, \tag{3.23}$$

where

$$x_c(t) = x_c(I_0(E) + I(t) - I(T), \phi_0 + \tilde{\Omega}t + \phi(t)). \tag{3.24}$$

is the periodic function:

$$x_c(I_0(E)+I(t)-I(T), (\phi_0+2\pi)+\tilde{\Omega}t+\phi(t)) = x_c(I_0(E)+I(t)-I(T), \phi_0+\tilde{\Omega}t+\phi(t)). \tag{3.25}$$

Now we can consider the cancelation of angular perturbations.

4 Cancelation of angular perturbations

Introducing the perturbation-generating operator into the integral over ϕ_0:

$$R(E) = 2\pi \int_0^\infty dT e^{\frac{1}{2i}\int_0^T dtdt'\Theta(t'-t)\hat{I}(t)\hat{e}_I(t')} \times$$
$$\times \int_0^\infty dI_0 \int_0^{2\pi} \frac{d\phi_0}{\Omega(E)} e^{\frac{1}{2i}\int_0^T dtdt'\Theta(t'-t)\hat{\phi}(t)\hat{e}_\phi(t')} e^{-iV_T(x_c,e_c)}, \tag{4.1}$$

the mechanism of cancellations of the angular perturbations becomes evident. One can formulate the statement:

(i) if

$$e^{\frac{1}{2i}\int_0^T dtdt'\Theta(t'-t)\hat{\phi}(t)\hat{e}_\phi(t')}e^{-iV_T(x_c,e_c)} = e^{-iV_T(x_c,e_c)}|_{e_\phi=\phi=0} + dF(\phi_0)/d\phi_0, \tag{4.2}$$

and

(ii) if

$$F(\phi_0+2\pi) = F(\phi_0), \tag{4.3}$$

then we easily find:

$$R(E) = 2\pi\int_0^{2\pi}\frac{d\phi_0}{\Omega(E)}\int_0^\infty dTdI_0 e^{\frac{1}{2i}\int_0^T dtdt'\Theta(t'-t)(\hat{I}(t)\hat{e}_I(t')}e^{S(x_c+e\partial x_c/\partial\phi_0)-S(x_c-e\partial x_c/\partial\phi_0)}. \tag{4.4}$$

For the $(\lambda x^4)_1$-model

$$S(x_c+e\partial x_c/\partial\phi_0) - S(x_c-e\partial x_c/\partial\phi_0) = S_0(x_c) - 2\lambda\int_0^T dtx_c(t)\{e\partial x_c/\partial\phi_0\}^3, \tag{4.5}$$

where [6]

$$S_0(x_c) = \oint_T dt(\frac{1}{2}\dot{x}_c^2 - \frac{\omega_0^2}{2}x_c^2 - \frac{\lambda}{4}x_c^4) \tag{4.6}$$

is the closed time-path action and

$$x_c(t) = x_c(I(E)+I(t)-I(T),\phi_0+\tilde{\Omega}t). \tag{4.7}$$

(here $I(t)$ and $I(T)$ are the auxiliary variables). In this case the problem is quasiclassical over the angular degrees of freedom.

The condition (4.3) requires that the classical trajectory x_c, with all derivatives over I_0, ϕ_0, is the periodic function. In the considered case of $(\lambda x^4)_1$-model x_c is periodic function with period $1/\Omega$ [9], see (5.2). Therefore, we can concentrate our attention on the condition (4.2) only.

Expanding $F(\phi_0)$ over λ:

$$F(\phi_0) = \lambda F_1(\phi_0) + \lambda^2 F_2(\phi_0) + ... \tag{4.8}$$

we find frof (**??**) that

$$\frac{d}{d\phi_0}F_1(\phi_0) = \int_0^T \prod_{k=1}^3 dt'_k\hat{\phi}(t'_k)(-\frac{6}{(2i)^3})\int_0^T dt\prod_{k=1}^3\Theta(t-t'_k)x_c(t)(\partial x_c/\partial I_0)^3 e^{iS_0(x_c)} =$$

$$= \int_0^T dt'\hat{\phi}(t')\{(-\frac{6}{(2i)^3})\int_0^T dt\Theta(t-t')\prod_{k=1}^2(\Theta(t-t'_k)\hat{\phi}(t'_k))x_c(t)(\partial x_c/\partial I_0)^3 e^{iS_0(x_c)}\} \equiv$$

$$\equiv \int_0^T dt'\hat{\phi}(t')B_1(\phi). \tag{4.9}$$

This example shows that the sum over all powers of λ can be written in the form:

$$\frac{d}{d\phi_0}F(\phi_0) = \int_0^T dt'\hat{\phi}(t')B(\phi), \tag{4.10}$$

where, using the definition (3.20),

$$B(\phi) = \int_0^T dt\tilde{B}(\phi_0 + \phi(t)). \tag{4.11}$$

Therefore,

$$\hat{\phi}(t')B(\phi) = \frac{d}{d\phi_0}\int_0^T dt\delta(t-t')\tilde{B}(\phi_0 + \phi(t)) \tag{4.12}$$

is coincide with the total derivative over initial phase ϕ_0, and

$$F(\phi_0) = \tilde{B}(\phi_0 + \phi(t))|_{\phi=0}. \tag{4.13}$$

This result ends the consideration. It assumes that the expansion over interaction constant λ exist. Indeed, it is known [5] that the perturbation series for $(\lambda x^4)_1$-model with $\lambda > 0$ is convergent in Borel sense.

5 Conclusion

1. It was shown that the real-time quantum problem can be qusiclassical over the part of the degrees of freedom and quantum over another ones. Following to the result of this paper one may introduce the (probably naive) interpretation of the quantum systems integrability (we suppose that the classical system is integrable and can be mapped on the compact hypersurface in the phase space [11]): the quantum system is strictly integrable in result of cancelation of all quantum degrees of freedom. The mechanism of cancelation of the quantum corrections is varied from case to case.

For some problems (as the rigid rotator, or the Pocshle-Teller) the cancelation of quantum angular degrees of the freedom is enough since they carry only the angular ones. In an another case (as in the Coloumb problem, or in the one-dimensional models) the problem may be partly integrable since the quantum fluctuations of action degrees of freedom just survive. Theirs absence in the Coloumb problem needs special discussion (one must take into account the dynamical (hidden) symmetry of Coloumb problem [3]; to be published).

The transformation to the action-angle variables maps the N-dimensional Lagrange problem on the $2N$-dimensional phase-space torus. If the winding number on this hypertorus is a constant (i.e. the topological charge is conserved) one can expect the same cancellations. This is important for the field-theoretical problems (for instance, for *sine*-Gordone model [12]).

2. In the classical mechanics the following approximated method of calculations is used [11]. The canonical equations of motion:

$$\dot{I} = a(I,\phi), \qquad \dot{\phi} = b(I,\phi) \tag{5.1}$$

are changed on the averaged equations:

$$\dot{J} = \frac{1}{2\pi}\int_0^{2\pi} d\phi a(J,\phi), \qquad \dot{\phi} = b(J,\phi), \tag{5.2}$$

It is possible if the periodic oscillations can be extracted from the systematic evolution of the degrees of freedoms.

In our case

$$a(I,\phi) = j\partial x_c/\partial\phi, \qquad b(I,\phi) = \Omega(I) - j\partial x_c/\partial I. \tag{5.3}$$

Inserting this definitions into (5.2) we find evidently wrong result since in this approximation the problem looks like pure quasiclassical for the case of periodic motion:

$$\dot{J} = 0, \qquad \dot{\phi} = \Omega(J). \tag{5.4}$$

The result of this paper was used here. This shows that the procedure of extraction of the periodic oscillations from the systematic evolution is not trivial and this method should be used carefully in the quantum theories. (This approximation of dynamics is "good" on the time intervals $\sim 1/|a|$ [11].)

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